

# A NONEQUILIBRIUM THEORY OF SURFACE DEPOSITION FROM PARTICLE-LADEN, DILUTE CONDENSIBLE VAPOR-CONTAINING LAMINAR BOUNDARY LAYERS

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**Abstract**—The deposition rate of a condensible substance from, say, flowing combustion products to “cold” solid surfaces can be strongly influenced by the simultaneous presence of a particulate aerosol since the particles can: (a) “scavenge” vapor, thereby influencing the vapor deposition rate; and (b) thermophoretically drift to the cold surface, carrying their inventory of scavenged condensate. A rational, yet quite tractable thermophysical model of these nonequilibrium processes is developed here for high Reynolds number laminar stagnation region boundary layer flow, and implemented to the point of calculating and displaying the effects of mainstream particle loading, vapor loading and particle size on the deposition rate of condensible material at surface temperatures well below the vapor dew point. Despite the complexity of this multiphase flow situation, our theoretical model is quite general and is cast in terms of dimensionless parameters which dictate the importance of vapor-phase scavenging and particle thermophoresis, as well as the Kelvin (surface tension) effect in modifying the submicron particle (free-molecule) growth law. Illustrative numerical results are displayed for the deposition of alkali sulfate-like vapor from the combustion products of hydrocarbon fuel (or coal) with air, including the interesting “structure” of such nonequilibrium multiphase boundary layers. As a useful by-product, our results reveal which combinations of particle-phase parameters cause (a) previous “uncoupled” vapor/particle deposition rates to be approximately valid as well as (b) recent local vapor/condensate equilibrium limit results to be sufficiently accurate. We conclude with an outline of straightforward extensions of the present theory to include such factors as: (i) a nonuniform (polydispersed) particle size mainstream aerosol; and (ii) size-dependent particle thermophoretic diffusivity; which are likely to be important in current or future engineering applications.

## 1. INTRODUCTION—MOTIVATION

The performance of engineering equipment exposed to high-temperature gases containing both suspended particulate matter (“dust”) and condensible vapors is determined, in part, by *deposition* phenomena, which can lead to fouling and/or corrosion (e.g. Rosner & Atkins 1983; Rothman 1985). Even relatively recent work in the areas of submicron *particle* deposition (e.g. Goren 1977; Walker *et al.* 1979; Gökoğlu & Rosner 1985; Rosner & Kim 1984) and *vapor* deposition (Rosner *et al.* 1979) has, for simplicity, imagined that these processes are separable, i.e. noninteractive. However, very recently we have developed a quantitative theoretical explanation of experimentally observed deposition rate reductions experienced by highly cooled surfaces due to near-*equilibrium* condensation processes occurring *within* the gas near the deposition surface (Castillo & Rosner 1988a–c; Liang *et al.* 1988). Actually, in most engineering applications it is not immediately evident whether vapor/condensate equilibrium (VCE) *can* be achieved within such two-phase boundary layers (BLs). A more comprehensive theory of such situations would not only yield explicit dimensionless criteria for “nearness to VCE”, but would also enable the calculation of fully nonequilibrium vapor/particle transport situations—including the important effect of suspended aerosol particles on the rate at which condensible material arrives at the cooled surface (Rosner & Liang 1988). Our goal here is to develop a rational, but quite general and tractable mathematical model for accomplishing these seemingly ambitious goals.

### 1.1. Outline of the Present Paper

In this paper we present a tractable nonequilibrium theory of the deposition on “cold” surfaces of unary dilute vapors in high Reynolds number two-dimensional stagnation flow configurations.

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Of particular interest are conditions such that the vapor becomes supersaturated within the laminar thermal BL and a "monodispersed" (single-size) group of fine particles (perhaps impurities which serve as condensation nuclei) scavenge vapor in their transit across the supersaturated portion of the thermal BL. We demonstrate the importance of thermophoretic forces that drive particles (and the material condensed on them) toward the cold surface, thereby contributing to the total deposition rate of condensible material. The assumptions underlying our nonequilibrium, two-phase theoretical model and its ancillary basic equations are first enumerated in section 2. In section 3, our analytical and numerical methods are presented. Representative numerical results are then given in section 4. In section 5 some model generalizations are indicated. Section 6 summarizes our main conclusions and their implications with regard to the design and interpretation of experiments underway in our laboratory (Rosner & Liang 1988), and for operating equipment.

## 2. THERMOPHYSICAL MODEL AND GOVERNING EQUATIONS

### 2.1. Underlying Assumptions

To simplify the mathematical problem and at the same time to retain the most important characteristics of the system, the following assumptions (discussed further in section 4) will be made:

**A.1.** We restrict ourselves here† to systems in which the total amount of condensible material and solid particles is very small with respect to the amount of noncondensable (host) gas. Therefore, the velocity and temperature fields are not affected by the different processes (condensation, deposition etc.) the relatively small amounts of condensible material and particles undergo.

**A.2.** We assume steady laminar flow and make the usual BL approximations. Moreover, to eliminate the need to deal with coupled partial differential equations, self-similarity will be assumed (e.g. Schlichting 1968). In the problem of concern to us here, forced convection dominates buoyancy-induced convection (which is neglected).

**A.3.** Thermophysical properties of the gas (e.g. momentum and thermal diffusivities) will be considered constant and equal to their values for the carrier gas at mainstream condition (see, for example, section 4). Also, transport properties for the vapor (diffusion coefficient) and particles (thermophoretic coefficient) will be taken as constant. Lastly, the total system will be considered incompressible, i.e. the total density taken as approximately constant.

**A.4.** Thermal diffusion (Soret effect) is neglected for the vapor (Rosner 1980). Thus, the difference between the carrier gas and the vapor velocities is assumed to be only due to the diffusion flux of the vapor down its own concentration gradient. Also, we assume that the condensible vapor behaves as an ideal gas.

**A.5.** Solid particles do not exhibit appreciable Brownian diffusion and the difference between their velocity and the carrier gas velocity is dominated by the thermophoretic velocity of the particles. At each position within the BL this thermophoretic velocity will be equated to that corresponding to an isolated particle in a uniform gas flow with the same local temperature gradient, that is (e.g. Talbot 1981):

$$v_T = \alpha_T D_p \left[ \frac{-(\text{grad } T)}{T} \right], \quad [1]$$

where  $\alpha_T$  is a dimensionless "thermal diffusion factor". Actually,  $D_p$  is included here just to emphasize the similarity between [1] and a diffusion velocity. In fact, the value of the "thermophoretic diffusivity"  $\alpha_T D_p$  does not really depend on  $D_p$  (which, in fact, is taken to be zero in our analysis); rather, it depends on the gas momentum diffusivity, particle Knudsen number and particle- and carrier gas-thermal conductivities; being  $\alpha_T D_p \simeq 0.54\nu$ , irrespective of the particle radius and thermal conductivity for particles whose radius is much smaller than the mean-free-path of the gas molecules (Talbot 1981). This remarkably simple limiting case provides a useful first approximation for the magnitude of  $\alpha_T D_p$ , and motivates our introduction of the dimensionless parameter  $\alpha \equiv \alpha_T D_p / \nu$  in the analysis that follows.

†Heavily loaded multiphase BLs, including thermophoretic mass transport, of importance in the manufacture of optical wave guides, have recently been treated by Rosner & Park (1988; Park & Rosner 1988; Park 1987).

**A.6.** When the dispersed particles are in a region in which the vapor is supersaturated, each is able to capture vapor in accord with the free-molecular-regime law:

$$\dot{r} = \frac{\alpha_m \rho}{\rho_L} \left( \frac{RT}{2\pi M_v} \right)^{1/2} \cdot [\omega_v - \omega_v^{eq}(T, r)] \quad [2]$$

where  $\dot{r}$  is the growth rate of particle radius associated with vapor capture;  $\alpha_m$  is the mass accommodation (condensation) coefficient (at most, unity);  $\rho$  is the prevailing gas density; and  $\rho_L$  is the density of the condensed material (condensate particle mass divided by its volume).  $\rho_L$  should not be confused with the dispersed condensate “phase” density  $\rho_c$ ; i.e. the condensate mass per unit *total* volume (particles plus gas). In [2] we have implicitly used the assumption of condensible vapor ideality *via* the relation between local vapor mass fraction,  $\omega_v$ , and the partial vapor pressure  $p_v$ :

$$\omega_v = \left( \frac{1}{\rho} \right) \cdot \frac{p_v M_v}{(RT)} \quad [3]$$

$\omega_v^{eq}(T, r)$  is the vapor mass fraction at which a particle of radius  $r$  would be in equilibrium with the surrounding vapor. Using [3] and Kelvin’s equation, it is given by

$$\omega_v^{eq}(T, r) = \omega_v^{eq}(T, \infty) \cdot \exp \left( \frac{2\sigma M_v}{\rho_L RT r} \right), \quad [4]$$

where the multiplier  $\omega_v^{eq}(T, r = \infty)$  is the equilibrium vapor mass fraction over a flat condensate surface ( $r = \infty$ ) at temperature  $T$ . Taking into account [3] and the temperature dependence of the equilibrium vapor pressure over a flat layer of condensate (Clausius–Clapeyron), hereafter we use

$$\omega_v^{eq}(T, \infty) = \frac{\text{const}}{T} \cdot \exp \left( -\frac{\Lambda}{RT} \right), \quad [5]$$

where const and  $\Lambda$  are each specific to the vapor considered ( $\Lambda$  being the molar heat of vaporization).

## 2.2. Host (Carrier Gas) Flow Field

As indicated in section 1.2, we consider the two-dimensional stagnation point (Hiemenz) flow. This corresponds to a steady flow which arrives from the  $y$ -axis, impinges on a flat solid wall placed at  $y = 0$ , where it divides into two streams near the wall, leaving in both ( $\pm$ ) directions. The external (inviscid) velocity distribution in the neighborhood of the symmetrical forward stagnation “point” (at  $x = y = 0$ ) is given by (e.g. Schlichting 1968):

$$u_e(x) = \left( \frac{du_e}{dx} \right)_{x=0} \cdot x. \quad [6]$$

In the immediate vicinity of the solid wall, viscous (momentum diffusion) effects become important and, for a Newtonian fluid, the velocity field must satisfy the well-known two-dimensional BL equations:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \quad [7]$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad [8]$$

with boundary conditions:  $u = 0$  (no slip) and  $v = 0$  (no blowing) at  $y = 0$  and  $u = u_e(x)$  at  $y = \infty$ .

Introducing the “stretched” dimensionless ordinate

$$\eta \equiv y \left[ \frac{1}{\nu} \left( \frac{du_e}{dx} \right)_{x=0} \right]^{1/2} \quad [9]$$

and a stream function given by

$$\psi(x, y) = \left[ v \left( \frac{du_c}{dx} \right)_{x=0} \right]^{1/2} \cdot f(\eta) \cdot x \quad [10]$$

the equation of local mass conservation [8] is automatically satisfied and the velocity components become

$$u = \frac{\partial \psi}{\partial y} = u_c(x) \cdot f'(\eta) \quad [11]$$

and

$$v = -\frac{\partial \psi}{\partial x} = - \left[ v \left( \frac{du_c}{dx} \right)_{x=0} \right]^{1/2} \cdot f(\eta), \quad [12]$$

where above, and in what follows, primes denote differentiation with respect to  $\eta$ . Introducing these expressions into the  $x$ -momentum balance equation [7], the following well-known nonlinear third-order (Blasius) ODE for  $f(\eta)$  is obtained:

$$f''' + ff'' + [1 - (f')^2] = 0 \quad [13]$$

with the boundary conditions:

$$f = f' = 0 \quad @ \eta = 0 \quad [14]$$

$$f' = 1 \quad @ \eta = \infty. \quad [15]$$

Notice that our assumptions of constant thermophysical properties and low vapor/condensate mass loading allow  $f(\eta)$  to be determined independently of the temperature and mass-fraction fields discussed below. Indeed, we will make use of previous numerical computations of this well-known (Blasius) function (Schlichting 1968).

### 2.3. Temperature Field

In the steady state, using laminar BL approximations A.2 and A.3, the PDE which governs the temperature distribution  $T(x, y)$  is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_h \frac{\partial^2 T}{\partial y^2}, \quad [16]$$

$\alpha_h$  being the heat (thermal) diffusivity. Defining

$$\theta \equiv \frac{T}{T_\infty} \quad [17]$$

when the wall temperature,  $T_w$ , is held constant, the ODE for  $\theta(\eta)$  becomes

$$\theta'' + \text{Pr} \cdot f(\eta) \theta' = 0 \quad [18]$$

subject to the boundary conditions

$$\theta(0) \equiv \theta_w = \frac{T_w}{T_\infty}, \quad \theta(\infty) = 1, \quad [19]$$

where Pr is the host gas Prandtl number,  $v/\alpha_h$ . The solution can be written in the following quadrature form (e.g. Spalding & Evans 1961):

$$\theta(\eta) = \theta_w + (1 - \theta_w) \cdot \frac{1}{\delta_T} \int_0^\eta \exp \left[ -\text{Pr} \int_0^\varphi f(\xi) d\xi \right] d\varphi \quad [20]$$

where  $f(\eta)$  is defined by [13]–[15] and

$$\delta_T \equiv \int_0^\infty \exp \left[ -\text{Pr} \int_0^\varphi f(\xi) d\xi \right] d\varphi. \quad [21]$$

For a description of the computation of  $\delta_T$  see, for example, Castillo & Rosner (1988a), section 3.1. For  $Pr = 0.7$  (e.g. air) we find  $\delta_T = 2.01669$ .

### 2.4. Mass Fraction Equations

Now consider that in the mainstream there is an undersaturated condensible vapor with mass fraction  $\omega_{v,\infty}$  and  $N_{p,\infty}$  "foreign" particles per unit volume, each with the same radius,†  $r_\infty$ . If the wall temperature is sufficiently low the vapor will become supersaturated *within* the thermal BL and the particles will be able to scavenge some vapor in this saturated region in accord with the particle growth law given by [2]. Under these circumstances, the mass fraction of condensible material actually attached to the particles (relative to the total density) will be given by

$$\omega_c = \frac{\rho_L}{\rho} \cdot \frac{4}{3} \pi (r^3 - r_\infty^3) \cdot N_p. \quad [22]$$

Of course this condensed material will be transported with a velocity corresponding to the particles,  $\mathbf{v}_p = \mathbf{v} + \mathbf{v}_T$ ; the thermophoretic velocity,  $\mathbf{v}_T$ , being given by [1]. Under assumptions A.1–A.5, the coupled BL equations describing local conservation of condensible material in the vapor phase,  $\omega_v \equiv \rho_v/\rho$ , and in the dispersed condensate phase,  $\omega_c \equiv \rho_c/\rho$ , can be written (Castillo & Rosner 1988a):

$$u \frac{\partial \omega_v}{\partial x} + v \frac{\partial \omega_v}{\partial y} = D_v \cdot \frac{\partial^2 \omega_v}{\partial y^2} - \frac{\dot{r}'''}{\rho} \quad [23]$$

$$u \frac{\partial \omega_c}{\partial x} + (v + v_T) \frac{\partial \omega_c}{\partial y} = -\omega_c \frac{\partial v_T}{\partial y} + \frac{\dot{r}'''}{\rho}, \quad [24]$$

where  $\dot{r}'''$  denotes the mass of vapor locally scavenged by the particles per unit time and volume.

The local number density of particles,  $N_p$ , must also satisfy an equation similar to [24] but without the source term; i.e. in the absence of particle coagulation‡ or break-up  $\text{div} \{(\mathbf{v} + \mathbf{v}_T) \cdot N_p\} = 0$  so that:

$$u \frac{\partial N_p}{\partial x} + (v + v_T) \frac{\partial N_p}{\partial y} = -N_p \frac{\partial v_T}{\partial y}. \quad [25]$$

The particles grow due to the capture of vapor along their own streamlines. Thus,

$$\dot{r} = \mathbf{v}_p \cdot \mathbf{grad} r, \quad [26]$$

where  $\dot{r}$  (local state) is given by [2]. Using [1] and the results of sections 2.2 and 2.3 we find

$$\mathbf{v}_p \cdot \mathbf{grad} r = - \left( \frac{du_c}{dx} \right)_{x=0} \cdot \left( f + \frac{\alpha}{\theta} \cdot \frac{d\theta}{d\eta} \right) \cdot \frac{dr}{d\eta}. \quad [27]$$

It should be mentioned that if more general "wedge-flows" were considered [recall that two-dimensional stagnation point flow is a particular case of the steady flow of a fluid past a wedge of opening angle  $\pi\beta$  (Schlichting 1968)], the requirement that the r.h.s. terms of [27] and [2] must be equal would exclude the possibility of self-similar solutions, except for the two-dimensional stagnation point flow. For this reason, according to assumption A.2, we have restricted ourselves to the latter case. Using [27] and [2] in [26] we obtain the interrelation

$$\left( f + \frac{\alpha}{\theta} \cdot \frac{d\theta}{d\eta} \right) \cdot \frac{dr}{d\eta} = - \frac{\rho}{\rho_L} \cdot \frac{\alpha_m}{\left( \frac{du_c}{dx} \right)_{x=0}} \cdot \left( \frac{RT}{2\pi M_v} \right)^{1/2} \cdot [\omega_v - \omega_v^{*q}(T, r)]. \quad [28]$$

Using the value of  $\omega_c$  given by [22], together with the equation for  $N_p$ , [25], the value of  $dr/d\eta$

†This assumption is readily relaxed in order to deal with a *distribution* of particle sizes in the mainstream (see section 5).  
‡In applications with high particle mass loading BL coagulation cannot be neglected (Park & Rosner 1988).

given by [28] and the results of sections 2.2 and 2.3 in [24], we find that the local condensate source strength is given by

$$\dot{r}''' = 4\pi r^2 \rho \alpha_m \left( \frac{RT}{2\pi M_v} \right)^{1/2} \cdot [\omega_v - \omega_v^{\text{eq}}(T, r)] \cdot N_p. \quad [29]$$

At this point it is convenient to rescale all dependent variables with respect to their values in the mainstream. Thus, defining  $n \equiv N_p/N_{p,\infty}$ ,  $h \equiv r/r_\infty$  and  $\Omega_v \equiv \omega_v/\omega_{v,\infty}$ , from [25] the first ODE for  $n$  takes the simple form

$$A \frac{dn}{d\eta} + Bn = 0. \quad [30]$$

Also, from the Eulerian form of the particle growth law, [28] becomes

$$A \cdot \frac{dh}{d\eta} = -C\theta^{1/2} \cdot [\Omega_v - \Omega_v^{\text{eq}}(\theta, h)]. \quad [31]$$

Taking into account the value of  $\dot{r}'''$  given by [29], from [23] we find that  $\Omega_v \equiv \omega_v/\omega_{v,\infty}$  must satisfy the second-order linear inhomogeneous ODE:

$$\frac{d^2\Omega_v}{d\eta^2} + \text{Sc} \cdot f \cdot \frac{d\Omega_v}{d\eta} = 3\text{Sc} \cdot CLnh^2\theta^{1/2} [\Omega_v - \Omega_v^{\text{eq}}(\theta, h)]. \quad [32]$$

In the above equations we have introduced the following dimensionless functions:

$$A(\eta) \equiv f(\eta) + \frac{\alpha}{\theta} \cdot \frac{d\theta}{d\eta} \quad [33]$$

and

$$B(\eta) \equiv -\frac{\alpha}{\theta} \cdot \frac{d\theta}{d\eta} \cdot \left( \text{Pr} \cdot f + \frac{1}{\theta} \frac{d\theta}{d\eta} \right); \quad [34]$$

and the dimensionless parameters

$$C \equiv \frac{\alpha_m}{r_\infty \left( \frac{du_e}{dx} \right)_{x=0}} \cdot \frac{\rho}{\rho_L} \cdot \left( \frac{RT_\infty}{2\pi M_v} \right)^{1/2} \cdot \omega_{v,\infty} \quad [35]$$

$$L \equiv \frac{\rho_L}{\rho} \cdot \frac{\frac{4}{3} \pi r_\infty^3 \cdot N_{p,\infty}}{\omega_{v,\infty}}. \quad [36]$$

Defining a dew point temperature,  $T_{\text{dp}}$ , such that

$$\omega_{v,\infty} = \omega_v^{\text{eq}}(T_{\text{dp}}, r = \infty), \quad [37]$$

we can write

$$\Omega_v^{\text{eq}}(\theta, h) = \Omega_v^{\text{eq}}(\theta, \infty) \cdot \exp\left(-\frac{K}{h\theta}\right) \quad [38]$$

with:

$$\Omega_v^{\text{eq}}(\theta, \infty) = \frac{\theta_{\text{dp}}}{\theta} \cdot \exp\left[\mathcal{L}\left(\frac{1}{\theta_{\text{dp}}} - \frac{1}{\theta}\right)\right]. \quad [39]$$

Here the parameters  $K$ ,  $\theta_{\text{dp}}$  and  $\mathcal{L}$  are defined by

$$K \equiv \frac{2\sigma M_v}{\rho_L RT_\infty r_\infty}, \quad [40]$$

$$\theta_{\text{dp}} \equiv \frac{T_{\text{dp}}}{T_\infty} \quad [41]$$

and

$$\mathcal{L} \equiv \frac{\Lambda}{(RT_\infty)}. \quad [42]$$

The parameter  $C$  is a Damköhler number (see Rosner 1986) related to the scavenging power of each particle,  $L$  is a measure of the relative mass loading of the foreign particles in the mainstream (calculated as if they had the density  $\rho_L$ ) and  $K$  is a measure of the importance of the Kelvin effect. When  $K \ll 1$ , the exponential term in [38] can be approximated by unity, i.e. the Kelvin effect (increase of equilibrium vapor pressure over a curved condensate surface) becomes negligible.

The system of equations [30]–[32] for  $n$ ,  $h$  and  $\Omega_v$  must be solved subject to the following boundary conditions:

$$\textcircled{a} \eta = 0 \quad \Omega_v = \Omega_v^{\text{eq}}(\theta_w, h = \infty) \tag{43}$$

$$\textcircled{a} \eta = \infty \quad \Omega_v = n = h = 1. \tag{44}$$

The boundary condition [43] establishes that the vapor at  $y = 0$  is at thermodynamic equilibrium with the material deposited on the wall. The solution of [30] can be written in the form of a quadrature:

$$n = \exp \left[ \int_{\eta}^{\infty} \frac{B(\varphi)}{A(\varphi)} \cdot d\varphi \right]. \tag{45}$$

Interestingly enough, this equation can be solved independently of the other two because we have assumed that the thermophoretic diffusivity parameter  $\alpha$  is insensitive to particle radius.

The normalized mass fraction of condensed material,  $\Omega_c = \omega_c/\omega_{v,\infty}$ , (locally attached to the particles) is given by

$$\Omega_c = L \cdot (h^3 - 1) \cdot n. \tag{46}$$

By assumption, the foreign particles do not sublime in the gas, therefore the above set of equations must be solved subject to the condition that  $h$  cannot be smaller than the mainstream value unity.

The local mass deposition rate of condensible material *in vapor form* onto the wall will be given by

$$-j''_{v,w} = \rho D_v \left( \frac{\partial \omega_v}{\partial y} \right)_{y=0} = \frac{\rho}{Sc} \cdot \left[ v \left( \frac{du_c}{dx} \right)_{x=0} \right]^{1/2} \cdot \omega_{v,\infty} \cdot \frac{d\Omega_v}{d\eta} \Big|_{\eta=0}, \tag{47}$$

a result independent of specific location along the wall. If we define a dimensionless *vapor* deposition rate  $\mathcal{J}_v$ , given by

$$\mathcal{J}_v \equiv \frac{1}{\rho \omega_{v,\infty}} \cdot \left[ v \left( \frac{du_c}{dx} \right)_{x=0} \right]^{-1/2} \cdot (-j''_{v,w}) \tag{48}$$

then, from [47],

$$\mathcal{J}_v = \frac{1}{Sc} \cdot \frac{d\Omega_v}{d\eta} \Big|_{\eta=0}. \tag{49}$$

If the particles have been able to capture some vapor in their transit across the thermal BL upon arrival at the wall, the condensed material attached to them, of course, contributes to the total deposition rate. Owing to our definition of  $\Omega_c$ , this contribution is given by

$$-j''_{c,w} = -(\rho_c v_T)_{y=0} = \rho \left[ v \left( \frac{du_c}{dx} \right)_{x=0} \right]^{1/2} \cdot \frac{1 - \theta_w}{\delta_T \theta_w} \cdot \alpha \omega_{v,\infty} \cdot \Omega_c(\eta = 0). \tag{50}$$

If we now define the dimensionless deposition rate of *condensed* material by

$$\mathcal{J}_c \equiv \frac{1}{\rho \omega_{v,\infty}} \cdot \left[ v \left( \frac{du_c}{dx} \right)_{x=0} \right]^{-1/2} \cdot (-j''_{c,w}) \tag{51}$$

then, using [46], we find

$$\mathcal{J}_c = \frac{1 - \theta_w}{\delta_T \theta_w} \cdot \alpha L [(h^3 - 1) \cdot n]_{\eta=0}. \tag{52}$$

In displaying our results for the structure of nonequilibrium two-phase BLs it will be useful to

display the local rate at which the particles scavenge vapor. For this purpose we introduce a nondimensional local vapor scavenging rate by the particles:

$$\dot{\Omega}_v''' \equiv \frac{\dot{r}'''}{\rho\omega_{v,\infty} \left( \frac{du_c}{dx} \right)_{x=0}}. \quad [53]$$

Using [29] we find

$$\dot{\Omega}_v'''(\eta) = 3CLnh^2\theta^{1/2} \cdot [\Omega_v - \Omega_v^{eq}(\theta, h)], \quad [54]$$

which is seen to be equal to the r.h.s. of [32] divided by the vapor/mixture Schmidt number,  $Sc$ .

### 3. ANALYTICAL RESULTS AND NUMERICAL METHODS

If we solve [32], with the boundary conditions for  $\Omega_v$  given by [43] and [44] and with the r.h.s. of the equation set equal to zero, and it turns out that  $\Omega_v < \Omega_v^{eq}(\theta, 1)$  everywhere, then clearly no vapor can be captured by the foreign particles in their transit across the BL (i.e.  $\dot{r}$  given by [2] is never positive). In this case, the solution obtained by setting the r.h.s. of [32] equal to zero, would be the actual solution. Under such circumstances particles and vapor each deposit on the wall, but they do not interact with each other. This noncondensation case has already been studied (e.g. Castillo & Rosner 1988a) but, for completeness, a brief indication of the solution is given in section 3.1.

The most interesting cases for the present study, in which  $\Omega_v$  becomes larger than  $\Omega_v^{eq}(\theta, 1)$  somewhere in the BL (corresponding to local "supersaturation" and the possibility of vapor scavenging), will be studied in section 3.2.

#### 3.1. No Boundary Layer Condensation Case

When the particles are unable to scavenge vapor, the normalized vapor mass fraction,  $\Omega_v$ , must satisfy [32] with the r.h.s. set equal to zero. The solution can be written in quadrature form (e.g. Castillo & Rosner 1988a):

$$\Omega_v = \Omega_{v,w} + (1 - \Omega_{v,w}) \cdot \Gamma(\eta, 0) \quad [55]$$

with

$$\Gamma(\eta, \phi) = \frac{1}{\delta_v(\phi)} \cdot \int_{\phi}^{\eta} \exp \left[ -Sc \int_{\phi}^{\xi} f(\xi) d\xi \right] d\phi \quad [56]$$

and

$$\delta_v(\phi) = \int_{\phi}^{\infty} \exp \left[ -Sc \int_{\phi}^{\xi} f(\xi) \cdot d\xi \right] d\phi. \quad [57]$$

Note that in the limit  $\phi = 0$ ,  $\delta_v(\phi)$  is equivalent to  $\delta_T$  with  $Sc$  replacing  $Pr$ . According to the boundary condition,

$$\Omega_{v,w} = \Omega_v^{eq}(\theta_w, h = \infty). \quad [58]$$

Thus, the dimensionless vapor deposition rate, given by [49], becomes

$$\mathcal{J}_v = \frac{1 - \Omega_v^{eq}(\theta_w, h = \infty)}{Sc \cdot \delta_v(0)}, \quad [59]$$

which can easily be calculated once the parameters  $\theta_{dp}$ ,  $\mathcal{L}$  and  $Sc$  are specified. It is clear from [39] that  $\mathcal{J}_v$  is negative for  $\theta > \theta_{dp}$  (if there exists a condensate layer on the wall it will evaporate at the rate given by [59]) and positive for  $\theta < \theta_{dp}$ ; i.e.  $T_{dp} = \theta_{dp} T_{\infty}$  represents the conventional dew point temperature.  $\delta_v(0)$  depends only on the vapor Schmidt number, and a description of the computation of  $\delta_v(0)$  for any value of  $Sc$  can be found in Castillo & Rosner (1988a).

#### 3.2. Condensation (Heterogeneous Nucleation) Within the Boundary Layer

Let us assume that at a certain position  $\eta_n$  inside the thermal BL  $\Omega_v = \Omega_v^{eq}(\theta, 1)$ , being  $\Omega_v < \Omega_v^{eq}(\theta, 1)$  for  $\eta > \eta_n$ , and the particles are unable to capture vapor in this region and  $\Omega_v \geq \Omega_v^{eq}(\theta, 1)$  for some value of  $\eta \leq \eta_n$  [note that due to the boundary condition at the wall,



$\Omega_{v,w} < \Omega_v^{eq}(\theta_w, h_w)$  (when  $K \neq 0$ ) and owing to the Kelvin effect the particles have to lose some of the captured vapor in the immediate vicinity of the wall]. For  $\eta > \eta_n$  the r.h.s. of [32] must be set equal to zero, and the solution is

$$\Omega_v = \Omega_{v,n} + (1 - \Omega_{v,n}) \cdot \Gamma(\eta, \eta_n), \quad \eta \geq \eta_n, \quad [60]$$

with  $\Gamma(\eta, \eta_n)$  given by [56] and  $\Omega_{v,n} = \Omega_v^{eq}(\theta_n, 1)$ .

Continuity of condensible material flux across the locus  $\eta = \eta_n$  implies that  $d\Omega_v/d\eta$  is continuous at  $\eta = \eta_n$  (Castillo & Rosner 1988a). Thus, from [60]:

$$\left. \frac{d\Omega_v}{d\eta} \right|_{\eta_n} = \frac{1 - \Omega_{v,n}}{\delta_v(\eta_n)}. \quad [61]$$

For analyzing conditions in the region  $\eta \leq \eta_n$ , we denote

$$X \equiv \Omega_v \quad [62]$$

and

$$Y \equiv \frac{d\Omega_v}{d\eta}. \quad [63]$$

Equations [31] and [32] can be transformed into the following ODE system:

$$X' = Y \quad [64]$$

$$Y' = -Sc \cdot f Y + 3 Sc \cdot C \cdot L [X - \Omega_v^{eq}(\theta, h)] n \theta^{1/2} h^2 \quad [65]$$

$$h' = -C \theta^{1/2} \cdot \frac{\{X - \Omega_v^{eq}(\theta, h)\}}{A}; \quad [66]$$

with boundary conditions at  $\eta_n$ :

$$X = \Omega_v^{eq}(\theta_n, 1) \quad [67]$$

$$Y = \frac{(1 - X)}{\delta_v(\eta_n)} \quad [68]$$

$$h = 1; \quad [69]$$

and the final condition at  $\eta = 0$ :

$$X = \Omega_v^{eq}(\theta_w, \infty). \quad [70]$$

Here  $\theta(\eta)$  will be given by [20] and the  $n(\eta)$  by [45]. The dimensionless deposition rate of condensible material in *vapor* form,  $\mathcal{J}_v$ , then becomes

$$\mathcal{J}_v = \frac{Y(\eta = 0)}{Sc}. \quad [71]$$

The contribution due to the material condensed on the foreign particles is

$$\mathcal{J}_c = (1 - \theta_w) \alpha \frac{L[(h_w^3 - 1)n_w]}{(\delta_T \theta_w)} \quad [72]$$

and the *total* dimensionless deposition rate of condensible material is then given by

$$\mathcal{J} = \mathcal{J}_v + \mathcal{J}_c. \quad [73]$$

It is interesting to note that at  $\eta = \eta_n$ ,

$$\frac{d}{d\eta} \left( \frac{\Omega_v}{\Omega_v^{eq}(\theta, 1)} \right)_{\eta=\eta_n} \leq 0 \quad [74]$$

(Otherwise  $\Omega_v$  would be larger than  $\Omega_v^{eq}(\theta, 1)$  for values of  $\eta > \eta_n$ ). Using [60], we find

$$\delta_v(\eta_n) \geq \frac{1 - \Omega_v^{eq}(\theta_n, 1)}{\left[ \frac{d\Omega_v^{eq}(\theta, 1)}{d\theta} \cdot \frac{d\theta}{d\eta} \right]_{\eta=\eta_n}}. \quad [75]$$

Therefore  $\eta_n$  must be larger than, or equal to, the smaller value of  $\eta$  for which the above inequality is satisfied.

On the other hand, in the limiting case when  $K = 0$  (and then  $\Omega_v^{eq}(\theta, h)$  does not depend on the particle size  $h$ ) condensation within the BL is only possible when

$$\frac{d}{d\eta} \left( \frac{\Omega_v}{\Omega_v^{eq}(\theta, \infty)} \right) \geq 0 \quad @ \eta = 0, \quad [76]$$

with  $\Omega_v(\eta)$  being the value when condensation is not allowed, given by [55]. For  $K > 0$ , [76] is a necessary, but not sufficient, condition for BL condensation.

The following method of solution has been used: the function  $f(\eta)$  is well-known [see, for example, the tabulation in Schlichting's (1968) book]. When the value of  $f$  is required at a nontabulated value of  $\eta$ , it was calculated using a local Taylor expansion using the values of  $f, f', f''$  and the corresponding value of  $f'''$  from [13] at the closest tabulated value of  $\eta$ . The values of  $\theta(\eta)$ ,  $d\theta/d\eta$  and  $n(\eta)$ , when necessary, were obtained from [20] and [45] with the integrals calculated using Simpson's algorithm.

Our procedure to obtain the deposition rates, for given  $\theta_w$  and mainstream conditions, is as follows:

1. If the inequality [76] is not satisfied, BL condensation is not possible and the dimensionless deposition rate,  $\mathcal{J}_v$ , is simply calculated from [59].
2. Otherwise, the smaller value of  $\eta$  for which [75] is satisfied (with the equal sign) is calculated and initially assumed that this was the value of  $\eta_n$  at which condensation begins.
3. Given the value of  $\eta_n$ , system of first order ODEs [64]–[66] is then numerically integrated using a fourth-order Runge–Kutta method, starting at  $\eta = \eta_n$  (with initial conditions given by [67]–[69] and ending at the wall ( $\eta = 0$ )).
4. Once the wall is reached, the condition [70] is checked. It can be shown that  $X(\eta = 0)$  is an increasing function of the chosen value of  $\eta_n$ . Therefore, if  $X(\eta = 0) < \Omega_v^{eq}(\theta_w, \infty)$  a new larger value of  $\eta_n$  is chosen and Step 3 is repeated. If  $X(\eta = 0) > \Omega_v^{eq}(\theta_w, \infty)$  the numerical step used in the variation of  $\eta_n$  was reduced and the former value of  $\eta_n$  again considered coming back to Step 3. In the particular case when  $X(\eta = 0) > \Omega_v^{eq}(\theta_w, \infty)$  for the initial value of  $\eta_n$  given in Step 2, BL condensation is not possible (note that [76] provides a necessary, but not sufficient, condition for heterogeneous nucleation onset) and  $\mathcal{J}_v$  is given by [59]. Otherwise, Steps 3 and 4 are repeated until [70] is satisfied to the required precision.
5. For the value of  $\eta_n$ , for which [70] is satisfied, the value of  $\mathcal{J}_v$  is calculated from [71] and  $\mathcal{J}_c$  from [72]. The total dimensionless deposition rate under these conditions is then the sum:  $\mathcal{J}_c + \mathcal{J}_v$ . The absolute mass deposition flux (e.g. mg/cm<sup>2</sup> s) corresponding to this dimensionless sum is then calculated by multiplication with the reference mass flux  $[v(du_c/dx)_{x=0}]^{1/2} \cdot \rho\omega_{v,\infty}$  appropriate to the application of interest.

#### 4. RESULTS AND DISCUSSION

Our main concern here is the influence on deposition rate of the three mainstream particle-related parameters:  $C$ ,  $L$  and  $K$ . It is clear from [32] that the limit  $CL$  tending to zero (and also the limit  $K$  tending to infinity for any value of  $CL$ ) corresponds to situations in which the mainstream particles do not scavenge vapor at all and the corresponding solution will be given by the equations (re-)presented in section 3.1; in this limit denoted hereafter as frozen (sink-free) boundary layer (FBL), the only place vapor condensation occurs is on the macroscopic surface (target) located at  $\eta = 0$ . On the other hand, in the limit  $K = 0$  and  $CL$  tending to infinity, the solution of [32] is  $\Omega_v(\eta) = \Omega_v^{eq}(\theta(\eta), h = \infty)$ ; i.e. the local consumption rate of vapor in the region  $\eta \leq \eta_n$  is such that local thermodynamic equilibrium (between the vapor and the material condensed onto the drifting particles) is achieved. This case will denote the local thermal equilibrium boundary layer

(LTEBL) limit. The abovementioned limiting cases were recently studied by Castillo & Rosner (1988a,c) and the corresponding results will be included here for comparison purposes.

To simplify the presentation of our nonequilibrium results, all other system parameters will be fixed at appropriate characteristic values (Rosner *et al.* 1979). For applications, the most interesting cases are those in which the carrier gas is essentially air, therefore we have selected  $Pr = 0.7$ . Since we are mainly interested in the deposition of alkali sulfate vapors from gas turbine or coal combustion products with  $T_\infty \approx 1500$  K the value of  $\mathcal{L}$  defined by [42] is  $\approx 22$  (for  $Na_2SO_4$  vapors),  $\mathcal{L} \approx 19$  (for  $K_2SO_4$ ). Accordingly, for illustrative purposes we will use the value  $\mathcal{L} = 20$ . Also, a characteristic value of the Schmidt number for these dilute vapors in air is  $Sc = 1.8$  and in this case the value of the dimensionless FBL thickness  $\delta_v\{0\}$  defined by [57] is  $\delta_v\{0\} = 1.3991$ . The value of  $T_{dp}/T_\infty \equiv \theta_{dp}$  will be taken to be 0.8 and although for cooled walls  $T_w/T_\infty \equiv \theta_w$  could formally vary from 0 to 1, our assumption A.3 (quasi-constant thermophysical properties) will fail for values of  $\theta_w$  which are too small i.e. gas density changes associated with temperature differences would become important). Thus, the smallest considered value of  $\theta_w$  will be 0.5.† Lastly, with respect to the normalized particle thermophoretic diffusivity  $\alpha_T D_c/\nu \equiv \alpha$ , we will consider the value 0.1, which appears (Castillo & Rosner 1988a) to describe the experimentally studied deposition of  $Na_2SO_4$  condensate particles. However, we will also present some results for  $\alpha = 0.5$  (a value close to the theoretical maximum for the free-molecular regime) and for the lower value,  $\alpha = 0.01$ .

Figure 1 presents the predicted total deposition rate of condensible material on the wall as a function of the normalized wall temperature  $\theta_w$  for the abovementioned values of the parameters and the particular case  $C = L = 1$  for different values of the Kelvin parameter,  $K$ . The dashed line represents the corresponding deposition rate for the LTEBL-limiting case, and the dotted line gives the FBL limit. The vertical lines represent the separation between the noncondensation case (section 3.1) and condensation within the BL (section 3.2) that occurs at the lower wall temperatures. Note that an increase in the value of  $K$  induces a “delay” in the beginning of vapor condensation onto the particles inside the BL. The corresponding wall temperatures at which particles start to scavenge vapor with the BL are  $\theta_w = 0.670$  for  $K = 1$ ,  $\theta_w = 0.722$  for  $K = 0.1$  and  $\theta_w = 0.738$  for  $K = 0$ ; these values do not depend on the value of  $C$  or  $L$ . This phenomenon is due to the minimum saturation ratio,  $s = \Omega_v/\Omega_v^{eq}(\theta, h = 1)$  required for the particles to begin to scavenge vapor. From [38], this minimum value for scavenging is  $s_{min} = \exp(-K/\theta(\eta))$  and clearly increases with the Kelvin parameter  $K$ . Therefore this  $s_{min}$  has to be achieved within the BL for vapor scavenging to be accomplished by the particles. From its definition, [40],  $K \sim r_\infty^{-1}$  and, for instance, using the surface-tension data included in Janz *et al.* (1979), for  $Na_2SO_4$  at  $T_\infty = 1500$  K, the Kelvin parameter is about  $10^{-3}$  when  $r_\infty = 1 \mu m$  and of order unity for  $r_\infty = 10^{-3} \mu m$  [a particle size small enough for which, incidentally, assumption A.5 would fail due to the relative importance of Brownian (Fick) diffusion]. Also, with these data, we find  $C \approx 10^{15}/[r_\infty (du_e/dx)_{x=0}]$  and  $L \approx 10^9 N_{p,\infty} r_\infty^3$ , therefore  $L$  would be of order unity in the presence of  $10^3$  particles/cm<sup>3</sup> in the mainstream, while  $r_\infty \approx 1 \mu m$ . It is clear that even for  $K = 0.1$  the difference from the limiting case  $K = 0$ , is very small. Therefore it seems that neglect of the Kelvin effect (i.e. taking  $K = 0$ ) will be self-consistent and a good approximation for most cases of interest here.

Figure 2 shows the total deposition rate of condensible material on the wall for  $L = 1$ ,  $K = 0$  and different values of the single-particle scavenging rate (Damköhler) parameter  $C$ . Again, the dashed line represents LTEBL-limit results and the dotted line denotes FBL-limit results. Clearly, for very small values of  $C$  the deposition rates tend toward FBL-limit values because the scavenging power of the particles is very small; i.e. they are able to scavenge only a small amount of condensible vapor during their thermophoretic drift across the supersaturated region ( $\eta \leq \eta_n$ ). On the other hand, for larger values of  $C$  the results tend to LTEBL-limit values. It is interesting to observe that for some range of  $C$  and  $\theta_w$ , predicted deposition rates even exceed the LTEBL-limiting value. The explanation of this (remarkable) behavior will be given below in connection

†Gökoğlu & Rosner (1984) compared numerical results for heat and mass transfer in LBLs with and without property variations for vapors in air and proposed a simple correlation scheme. They showed that, in general, for the range  $0.25 \leq T_w/T_e \leq 4$  and for Lewis numbers  $(D/\alpha_i) < 1$  (as in our case), the results for mass transfer considering constant properties differ no more than 18% from the results using actual property variations. In particular, for  $K_2SO_4$  vapors in air, the difference was  $< 10\%$  for the same range of temperature ratios. Thus, even for the lower values of  $\theta_w$  we have considered the error we are making in the present analysis is acceptably small.

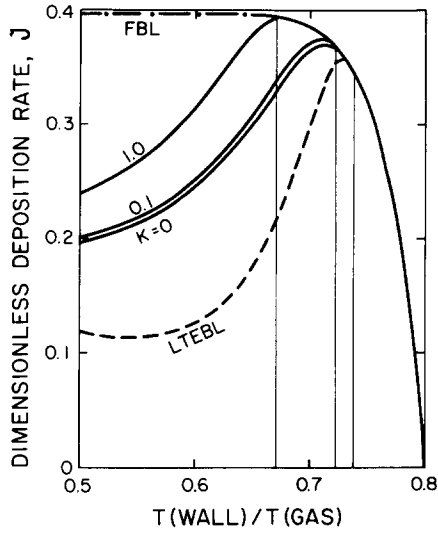


Figure 1. Total dimensionless deposition rate,  $\mathcal{J}$ , of condensable material on the cold surface as a function of the normalized wall temperature,  $\theta_w$ . The following conditions apply to all the figures presented in this paper:  $Pr = 0.7$ ,  $Sc = 1.8$ ,  $\mathcal{L} = 20$ ,  $\theta_{dp} = 0.8$ . For this figure, also:  $\alpha = 0.1$ ,  $C = 1$ ,  $L = 1$ . The dashed line corresponds to the LTEBL limit and the dotted line pertains to the FBL limit.

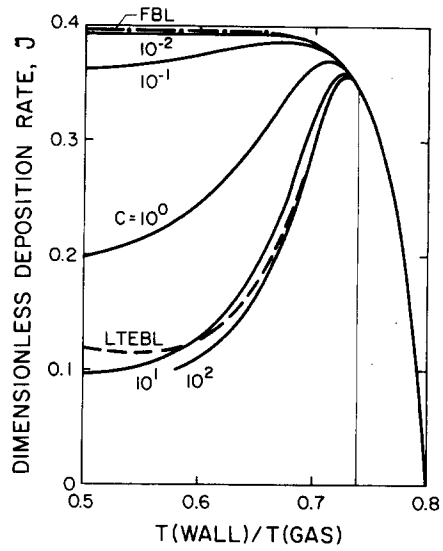


Figure 2. Surface temperature dependence of the total dimensionless deposition rate,  $\mathcal{J}$ , for  $\alpha = 0.1$ ,  $L = 1$ ,  $K = 0$  and various values of the particle scavenging efficiency (Damköhler) parameter  $C$ .

with our comments on figures 8 and 9. For large values of  $C$  and lower wall temperature ratios,  $\theta_w$ , the deposition rates cannot be formally obtained with this model for reasons given below. Indeed, the present model should be supplemented for these cases, as indicated in the appendix.

Figure 3 is equivalent to figure 2 but pertains to  $K = 1$ . The delay in the onset of condensation within the two-phase BL is indicated by the vertical lines. Again, for small values of  $C$  the total deposition rate tends to the FBL-limit values but, for larger values of  $C$ , tends to some limit that

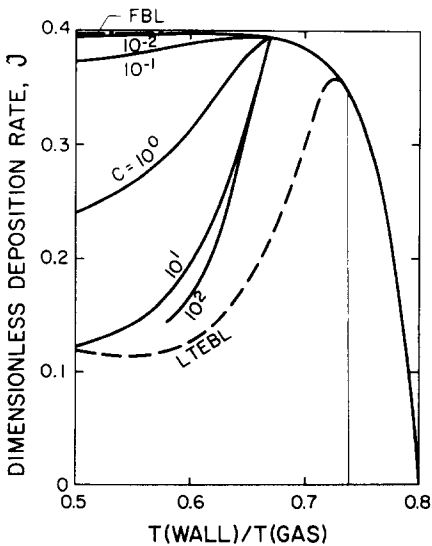


Figure 3. Surface temperature dependence of the total dimensionless deposition rate,  $\mathcal{J}$ , for  $\alpha = 0.1$ ,  $L = 1$ ,  $K = 1$  and various values of the particle scavenging efficiency (Damköhler) parameter  $C$ .

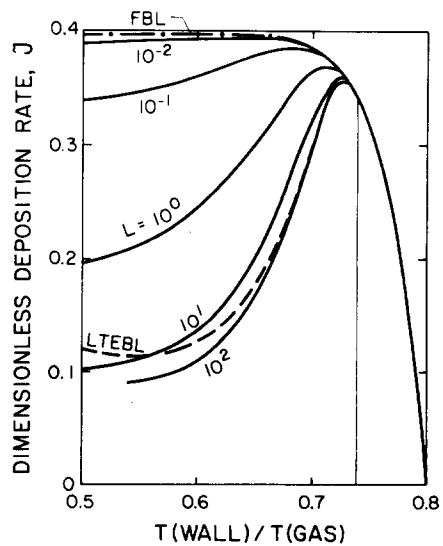


Figure 4. Surface temperature dependence of the total dimensionless deposition rate,  $\mathcal{J}$ , for  $\alpha = 0.1$ ,  $C = 1$ ,  $K = 0$  and different values of the particle loading parameter  $L$ .

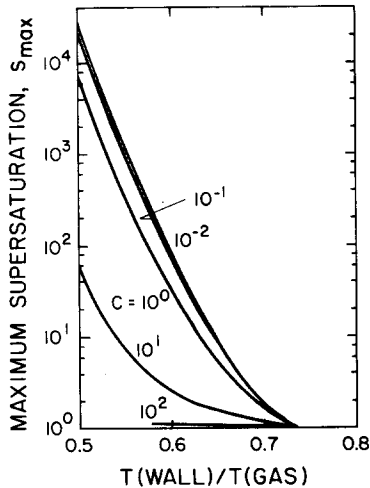


Figure 5. Maximum vapor saturation ratios achieved within the laminar BL for  $\alpha = 0.1$ ,  $L = 1$ ,  $K = 0$ .

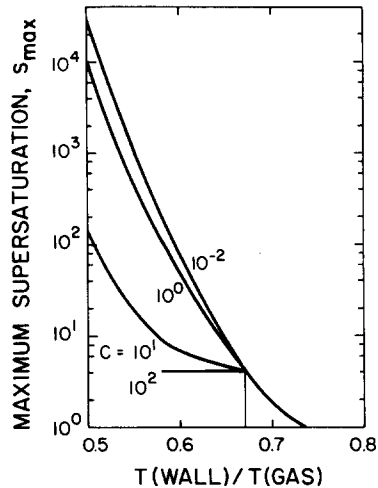


Figure 6. Maximum saturation ratios achieved within the laminar BL for  $\alpha = 0.1$ ,  $L = 1$ ,  $K = 1$ .

in this case does not correspond to LTEBL. This will be shown to be associated with the necessary build-up of  $s_{min}$  inside the BL, as indicated above when  $K \neq 0$ .

Similar results for  $C = 1$ ,  $K = 0$  and different values of the mainstream particle “loading” parameter  $L$  are plotted in figure 4. The trends parallel those indicated in figure 2, because, in fact, the abovementioned limits (FBL and LTEBL) are reached when the *product* CL tends to zero and infinity, respectively.

In the present model, condensation of vapor within the BL occurs *via* “heterogeneous” nucleation and subsequent growth onto the pre-existing, thermophoretically drifting particles. In view of the results presented in the previous figures, it is interesting to inquire if the scavenging power of the particles is sufficient to avoid the possibility of homogeneous nucleation. In figure 5 we plot the maximum value of the vapor saturation ratio,  $s = \Omega_v / [\Omega_v^{eq}(\theta, h = \infty)]$ , achieved within the BL for the conditions corresponding to figure 2 (i.e.  $L = 1$  and  $K = 0$ ). From the dew point temperature ratio  $\theta_w = 0.8$  down to the wall temperature at which condensation within the BL begins (located at  $\theta_w = 0.738$ ),  $s_{max} = 1$  and this value is achieved just and only at the wall, as required by the boundary condition [43]. For lower values of  $\theta_w$  and the represented values of  $C$ , the particles are not able to scavenge *all* “available” vapor and therefore a supersaturated region indeed appears within the BL, with the maximum value of  $s$  increasing as  $\theta_w$  and/or  $C$  decrease (recall that *at* the wall  $s = 1$ , as imposed by the LTE boundary condition). For the lower values of  $C$  the results tend to FBL-limit values and for larger values of  $C$ ,  $s_{max}$  departs slightly from unity [the LTEBL limit corresponds to a region of  $s = 1$  around the wall but  $s$  never exceeds unity; see figure 4.2-3 in Castillo & Rosner (1988a)]. To examine the self-consistency of our model, we must now consider the minimum supersaturation required for homogeneous nucleation to take place. Rosner *et al.* (1979) estimated this value (for  $\text{Na}_2\text{SO}_4$ -like species; see their figure 5.1) to be of the order of 10 for  $T = 1600$  K but  $s_{crit}$  could be above  $10^3$  when  $T = 1000$  K. Only when this minimum supersaturation for homogeneous nucleation is higher than the maximum saturation ratio achieved in the computed two-phase BL (figure 5) is homogeneous nucleation precluded and our present model self-consistent. In contrast, Ahluwalia & Im (1985) have presented a computational model in which the *only* condensation that takes place is *via* homogeneous nucleation† calculated

†However, their model is quite restrictive because, among other assumptions, they neglect the convective terms in the mass balance equations and they assume that once nucleated, particles *do not grow* because the vapor is very dilute. But the growth rate of a condensed particle depends on the local supersaturation level (and the latter even reaches the value  $10^8$  in some of their illustrative calculations). In a closely related, earlier analysis, Epstein & Rosner (1970) showed that once they are created, nucleated particles grow so rapidly that the rate of vapor consumption due to particle growth quickly becomes much more important than the contribution due to the generation of new particles.

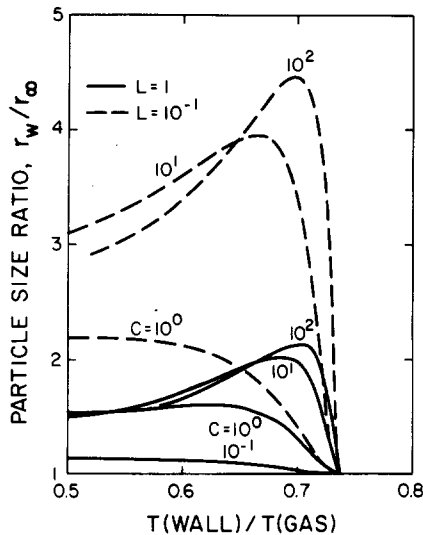


Figure 7. Normalized radius  $h = r/r_\infty$  of particles arriving at the wall for  $\alpha = 0.1$ ,  $K = 0$  and  $L = 1$  (solid lines), and  $L = 10^{-1}$  (dashed lines) for various values of the particle scavenging efficiency (Damköhler) parameter  $C$ .

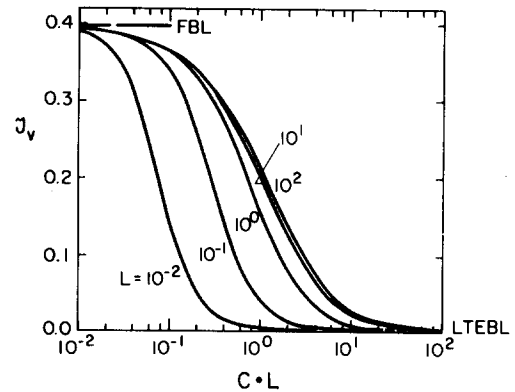


Figure 8. Dimensionless deposition rate of condensable material in vapor form,  $J_v$ , as a function of the scavenging rate parameter  $CL$  for  $\alpha = 0.1$ ,  $K = 0$  and a fixed wall temperature ratio,  $\theta_w = 0.6$ . The straight dashed lines in the upper-left and lower-right corners correspond, respectively, to the FBL and LTEBL limits.

according to classical nucleation theory. This phenomenon could, of course, be included in a generalization of the present heterogeneous condensation model when  $s_{\max} > (s_{\text{crit}})_{\text{hom nucl}}$ .

The maximum saturation ratio within the BL is also plotted in figure 6 for  $L = 1$ ,  $K = 1$  and values of the Damköhler group  $C$  corresponding to the deposition rates presented in figure 3. When the normalized wall temperatures drop from the dew point value,  $\theta_w = 0.8$ , to the value corresponding to onset of vapor condensation on the particles for  $K = 0$ ,  $\theta_w = 0.738$ ,  $s_{\max} = 1$  at the cold wall,  $\eta = 0$ . For lower  $\theta_w$  values, down to the onset of condensation for  $K = 1$  (at  $\theta_w = 0.670$ ),  $s_{\max} > 1$  somewhere within the BL (at a position that "leaves" the wall as  $\theta_w$  decreases) but is still lower than the minimum value required for condensation onto the particles [given by  $s_{\min} = \exp\{-K/\theta(\eta)\}$  corresponding to the situation in which  $\Omega_v$  has reached the value  $\Omega_v^{\text{eq}}(\theta, h = 1)$  and the pre-existing particles, with normalized radius  $h = 1$ , start to be able to scavenge vapor]. For even lower wall temperatures the particles scavenge as much vapor as they can, reducing the value of  $s_{\max}$  with respect to its value for an FBL. For the larger value of  $C$  considered,  $C = 10^2$ , the position at which  $s_{\max}$  is achieved almost coincides with the position at which condensation onto the particles begins (located at  $\eta = \eta_n$ , section 3.2). The line corresponding to  $C = 10^{-1}$  would not be distinguishable from the value for  $C = 10^{-2}$  on the scale used. The same reasoning concerning the possibility of homogeneous nucleation described in our discussion of figure 5 and the breakdown of the present model for this reason, is also applicable to figure 6.

Another assumption underlying the present model is that the normalized thermophoretic coefficient  $\alpha$  is insensitive to particle size. Talbot (1981) and others have shown that in the transition from the free-molecule regime to the continuum region,  $\alpha$  depends on the particle Knudsen number (equal to the ratio of the gas kinetic mean-free-path to the particle diameter). Therefore it is prudent to study the size of the condensate-enlarged particles arriving at the wall. This information is presented in figure 7 in which the normalized radius ( $h \equiv r/r_\infty$ ) of the particles reaching the wall is plotted as a function of the wall temperature for  $K = 0$ , and  $L = 1$  (solid lines) or  $L = 0.1$  (dashed lines), for different values of the Damköhler group  $C$ . Not surprisingly, from the dew point temperature,  $\theta_w = 0.8$ , down to the surface temperature at which particles can start to scavenge vapor,  $\theta_w = 0.738$ ,  $h_w = 1$ ; i.e. the particles arrive with their original size. For lower values of  $\theta_w$ , the "particle size upon arrival"  $h_w$ , increases as  $\theta_w$  decreases until a maximum value of  $h_w$  is reached. Thereafter, remarkably enough,  $h_w$  decreases. Anyway, the maximum increase in particle size (up to 4.5 times their original size) for the represented cases is not large enough to be a serious concern if the Knudsen number based on particle size at the wall is still large. Therefore, we conclude that

the assumption of a size-insensitive normalized thermophoretic diffusivity,  $\alpha$ , seems to be appropriate to our purpose. Actually, a dependence of  $\alpha$  on the particle radius (i.e.  $\alpha(h)$ ) could easily be introduced into a more comprehensive version of our model (see section 5).

As indicated in our comments on figure 2 (and also clear in figures 3 and 4) for larger values of the product  $CL$  the total deposition rate calculated formally as outlined earlier actually lies slightly below the value for the LTEBL limit and, for the lower values of  $\theta_w$ , our simple nonequilibrium model breaks down. This behavior can be explained with the help of figures 8 and 9, which depict the deposition rate in vapor form,  $\mathcal{J}_v$ , and attached to the particles (condensed form),  $\mathcal{J}_c$ , respectively, for a given wall temperature ratio  $\theta_w = 0.6$  when the Kelvin effect is neglected ( $K = 0$ ) as a function of the product  $CL$  for various values of the particle loading parameter  $L$ . In figure 8, the limiting values for  $\mathcal{J}_v$  are also represented for an FBL (upper left) and a LTEBL (lower right) by a dashed straight line. These vapor values are respectively,  $\mathcal{J}_v = 0.397$  and  $\mathcal{J}_v = 1.90 \times 10^{-3}$ . When the rate parameter  $CL$  is small, the amount of vapor scavenged by the particles is likewise small and  $\mathcal{J}_v$  tends to the FBL limit. On the other hand, for larger values of  $CL$  the results displayed have been terminated not for printing clarity but because the present model breaks down beyond a critical value of  $CL$  (which decreases as  $L$  decreases). The reason is the following:  $\mathcal{J}_v$  reaches the LTEBL limit at a finite value of  $CL$  and remains at this value for larger values of  $CL$ . But from its definition [49]  $\mathcal{J}_v \sim d\Omega_v/d\eta|_{\eta=0}$  and  $\mathcal{J}_v$  (LTEBL)  $\sim d\Omega_v^{eq}/d\eta|_{\eta=0}$ . Therefore, as soon as  $\mathcal{J}_v$  reaches the LTEBL limit,  $\Omega_v$  and  $\Omega_v^{eq}$  are tangent at  $\eta = 0$  and upon increasing the value of  $CL$  beyond that point (for  $L$  constant), a third region (not treated above) appears near the cold wall in which the particles are capable of taking more vapor than is locally available. Therefore in this third region (to be added to the two regions specified in section 3.2) LTEBL equilibrium is reached. Increasing the value of the particle scavenging efficiency parameter  $C$  (keeping the loading parameter  $L$  constant) this third region detaches from the wall and increases in thickness until at some value of  $C$  it engulfs the second region (in this second region the particles scavenge vapor but cannot consume all available vapor and  $s > 1$ ). This third region engulfs the second one and extends then from  $\eta_n$  (beginning of condensation, section 3.2, which, in fact, is a decreasing function of  $C$  and  $L$ ) to the wall reaching the LTEBL limit in the whole region in which the particles scavenge vapor (that is, in fact, the LTEBL limit that we have been talking about from the beginning). For this reason, as soon as  $\mathcal{J}_v$  reaches the LTEBL limit the present model must be improved. In the abovementioned third region  $\Omega_v = \Omega_v^{eq}$  and [31] and [32] should be rewritten. The analysis of this case, as an extension of the present model, is sketched in the appendix.

Figure 9 depicts the deposition rate in condensate form  $\mathcal{J}_c$  for a constant wall temperature ratio,  $\theta_w = 0.6$ . The FBL rate is, of course,  $\mathcal{J}_c = 0$  and the LTEBL rate is  $\mathcal{J}_c = 0.124$ , represented by the horizontal broken line on the r.h.s. of the graph. The trends are rather remarkable since for a constant value of  $L$ ,  $\mathcal{J}_c$  increases as  $C$  increases from the FBI value ( $\mathcal{J}_c = 0$ ) until it reaches a maximum and then decreases even below the LTEBL limit. The third region just mentioned, in which  $s = 1$  throughout, appears and self-consistent calculations cannot be continued with the present model (see the extension indicated in the appendix). It is expected that  $\mathcal{J}_c$  will grow again until it reaches the LTEBL value. The difference between the maximum value, the further minimum and the LTEBL limit increases as  $\theta_w$  decreases.

In figure 10, the important local fields are represented for  $\theta_w = 0.6$ ,  $C = 1$ ,  $L = 1$  and  $K = 0$ . The existence of the two different regions discussed in section 3.2 is displayed. For  $\eta \geq \eta_n$ , the BL is frozen and particles cannot scavenge vapor ( $h = 1$  throughout the region) and  $\Omega_v < \Omega_v^{eq}$ . At  $\eta_n$ ,  $\Omega_v = \Omega_v^{eq}$  and the particles start to take on some vapor during their thermophoretic drift through this nonequilibrium BL, increasing their size from  $h = 1$  at  $\eta = \eta_n$  to  $h_w = 1.590$ . The local rate of vapor consumption,  $\Omega_v''(\eta)$ , given by [54], is zero both at  $\eta_n$  and at the wall,  $\eta = 0$ , because at both locations there is equilibrium; i.e.  $\Omega_v = \Omega_v^{eq}$ . For this reason the slope of particle size variable  $h$  is zero at  $\eta_n$  as well as at the wall.  $\Omega_v > \Omega_v^{eq}$  for  $0 < \eta < \eta_n$ , leading to a saturation ratio which reaches the maximum value of 32.2 near the wall being  $s = 1$  at both extremes of the interval (recall that the value when  $\eta \rightarrow \infty$  (mainstream) is only  $s_\infty = 8.42 \times 10^{-3}$ ). Keeping the value of  $L$  constant and increasing the value of  $C$ , a third region would appear in the vicinity of the wall in which  $\Omega_v = \Omega_v^{eq}$  (i.e.  $s = 1$ ) and the vapor consumption is limited by the amount of vapor present rather than the scavenging power of the particles. In this case,  $\Omega_v''$  is positive at the wall as is the slope

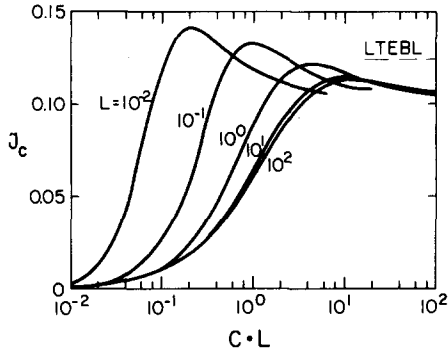


Figure 9. Dimensionless deposition rate of condensate material in condensate form (attached to the arriving particles),  $J_c$ , as a function of the scavenging rate parameter  $CL$  for various of  $L$  for  $\alpha = 0.1$ ,  $K = 0$  and a fixed wall temperature ratio  $\theta_w = 0.6$ . The straight dashed line on the upper r.h.s. corresponds to the LTEBL limit.

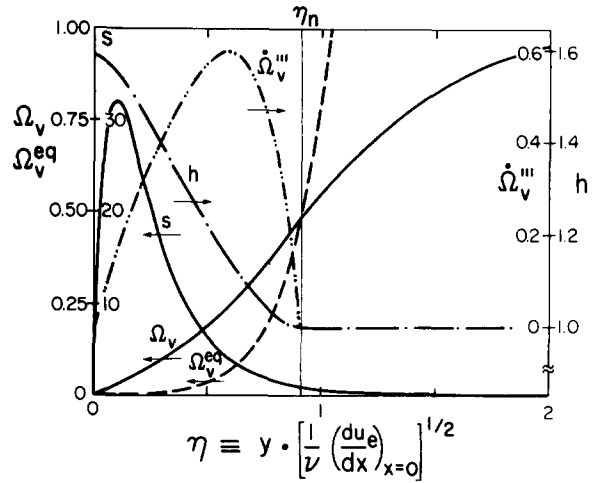


Figure 10. Predicted distribution of concentration fields for  $\alpha = 0.1$ ,  $C = 1$ ,  $L = 1$ ,  $K = 0$  and  $\theta_w = 0.6$ .

of  $h$ . On the other hand, for  $K > 0$ , due to the wall boundary condition  $\Omega_v = \Omega_v^{eq}(\theta_w, h = \infty)$ ,  $\dot{\Omega}_v'''$  and the slope of  $h$  are negative at the wall and, to the boundary condition, the particles actually give up a portion of their condensate to the vapor. This interesting effect is actually very small in most of the cases studied here.

Lastly, we present results for other values of the normalized particle thermophoretic diffusivity  $\alpha$ . Figure 11 depicts the total deposition rate for  $L = 1$ ,  $K = 0$  when  $\alpha = 0.01$  (cf. figure 2). The results are rather similar to those presented earlier for  $\alpha = 0.1$ . However, for  $\alpha = 0.5$  some remarkable behavior is observed, as indicated by figure 12. Whereas the FBL and LTEBL results are very close to each other for a certain range of  $\theta_w$  values, for the calculated range of  $C$  values the predicted deposition rate is always smaller than the FBL value, and behaves in the manner very similar to the case  $\alpha = 0.1$ . It is expected that by increasing the value of the particle scavenging efficiency parameter  $C$  beyond the values considered here, these trends will reverse. The correspond-

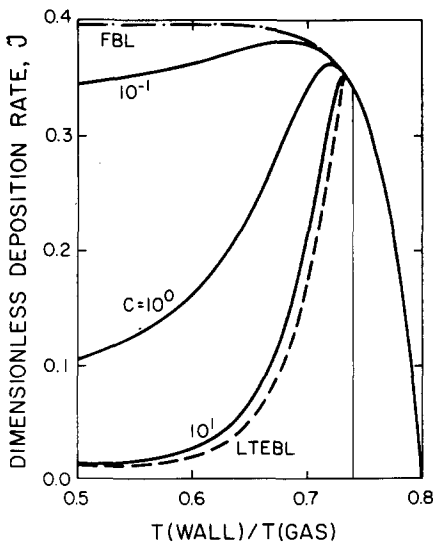


Figure 11. Surface temperature dependence of the total dimensionless deposition rate,  $J$ , for various values of the vapor scavenging efficiency (Damköhler) parameter  $C$  for  $L = 1$ ,  $K = 0$ ,  $\alpha = 0.01$ .

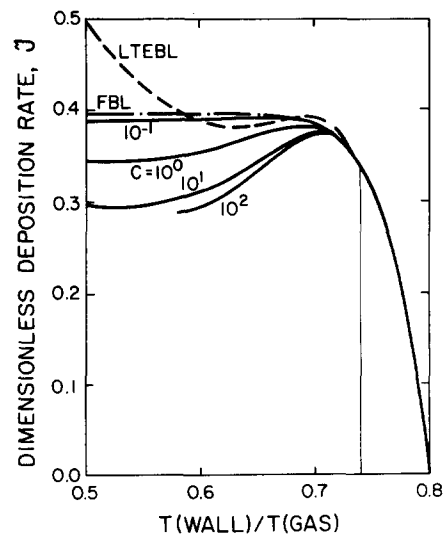


Figure 12. Surface temperature dependence of the total dimensionless deposition rate  $J$  for various values of the Damköhler parameter  $C$  for  $L = 1$ ,  $K = 0$ ,  $\alpha = 0.5$ .



ing total deposition rate will increase as  $C$  increases for a given value of  $\theta_w$ , upon the appearance of the equilibrium region (see the appendix), until the LTEBL limit is reached.

### 5. GENERALIZATIONS OF THE PRESENT MODEL

Some of the underlying assumptions specified in section 2.1 and used to generate the results discussed in section 4, can be relaxed, if necessary, leading to certain generalizations of the model that do not cause an excessive increase in complexity.

In the present model the particle *concentration* field is not affected by the vapor field (the converse is not true). This is due to our assumption that the normalized thermophoretic coefficient  $\alpha$  is insensitive to particle radius. Thus, once the normalized wall temperature is known, the particle density number is given by [45] and it is not affected by the amount of vapor condensing on the particles (through changes in particle size). As indicated in figure 7, depending on the values of  $C$ ,  $L$  and  $\theta_w$ , the particle radius can increase to almost five times its original value and in the transition (free-molecule to continuum) range of the particle Knudsen number the associated change in  $\alpha_\tau$  can be important (Talbot 1981). Thus, the treatment of scavenging by larger particles in BLs at higher pressures could be accommodated by introducing an  $\alpha(h)$  dependence. In such cases the particle concentration field would couple to the vapor field via  $\alpha(h)$  and the only thing that would change in the formulation above is the value of the function  $B(\eta)$ , which would now be

$$B(\eta) = -\frac{\alpha}{\theta} \frac{d\theta}{d\eta} \cdot \left( \text{Pr} \cdot f + \frac{1}{\theta} \frac{d\theta}{d\eta} \right) + \frac{1}{\theta} \frac{d\theta}{d\eta} \frac{d\alpha}{dh} \cdot \frac{dh}{d\eta}.$$

Using [31] this may be rewritten as

$$B(\eta) = -\frac{\alpha}{\theta} \frac{d\theta}{d\eta} \cdot \left( \text{Pr} \cdot f + \frac{1}{\theta} \frac{d\theta}{d\eta} \right) - \frac{C}{A(\eta)} \cdot \theta^{-1/2} \frac{d\theta}{d\eta} \frac{d\alpha}{dh} [\Omega_v - \Omega_v^{\text{eq}}(\theta, h)],$$

with  $A(\eta)$  still given by [33]. As indicated at the beginning of section 3, the added r.h.s. terms in  $B(\eta)$  should be set equal to zero for  $\eta \geq \eta_n$  (where particles do not scavenge vapor). Although the particle number density can still be formally written as in [45], with the new definition of  $B(\eta)$  and for  $\eta \leq \eta_n$  it is now affected by the vapor distribution through the second r.h.s. term of  $B(\eta)$  for  $\eta \leq \eta_n$ . Moreover, the equation for  $n$ , [30], should be added to the system of equations [64]–[66] with the boundary condition

$$n(\eta_n) = \exp \left[ \int_{\eta_n}^{\infty} \frac{B_0(\varphi)}{A(\varphi)} \cdot d\varphi \right],$$

with  $B_0(\eta)$  equal to the value of  $B(\eta)$  given by [34].

Another assumption in the present model is that all particles entering the BL have the same size. The more general case in which the particles in the mainstream are distributed among, say, discrete sets of particle sizes, can easily be accommodated. Let us assume there are  $N_{p,i}$  ( $i = 1, \dots, N$ ) particles with radius  $r_i$ , respectively. Defining  $n = N_{p,i}/N_{p,i}(\gamma \rightarrow \infty)$  and  $h_i = r_i/r_{i,\infty}$  the differential equation for  $n$  (the same for all particle sizes) is still [30] and its formal solution is given by [45]. The equation for each  $h_i$  is

$$A(\eta) \frac{dh_i}{d\eta} = -C_i \theta^{1/2} \cdot [\Omega_v - \Omega_v^{\text{eq}}(\theta, h_i)],$$

with the obvious definition of  $C_i$  (with respect to the particle size  $r_{i,\infty}$ ). The only other required change would be in [32] for  $\Omega_v$ , which would generalize to

$$\frac{d^2 \Omega_v}{d\eta^2} + \text{Sc} \cdot f \cdot \frac{d\Omega_v}{d\eta} = 3\text{Sc} \cdot n \cdot \theta^{1/2} \sum_{i=1}^N C_i L_i h_i^2 [\Omega_v - \Omega_v^{\text{eq}}(\theta, h_i)],$$

(with  $L_i$  given by [36] but referring to the particle size class  $r_{i,\infty}$ ). The value of  $\Omega_v^{\text{eq}}(\theta, h_i)$  is

$$\Omega_v^{\text{eq}}(\theta, h_i) = \Omega_v^{\text{eq}}(\theta, \infty) \cdot \exp \left( \frac{K_i}{h_i \theta} \right).$$

Note that when each Kelvin parameter  $K_i$  is very small (and the exponential term can be approximated by unity) for all particle sizes, the term  $[\Omega_v - \Omega_v^{eq}(\theta, h_i)]$  would not depend on  $h_i$  and could be taken out of the summation in the above equation for  $\Omega_v$ . In such cases, it is clear and intuitively satisfying that the parameter determining the important limiting behavior in the vapor scavenging becomes  $\sum_{i=1}^N C_i L_i$ . Thus, deposition rates will tend to the FBL limit when  $\sum_{i=1}^N C_i L_i$  tends to zero and to the LTEBL limit when  $\sum_{i=1}^N C_i L_i$  is very large.

## 6. CONCLUSIONS

By exploiting recent developments in the understanding of particle thermophoretic drift across gas BLs we have developed and demonstrated a self-consistent yet tractable theory for predicting the effect of a mainstream particulate aerosol on the deposition of condensible material to a solid surface maintained well below the vapor dew point. Our thermophysical model is free of the limiting assumptions which have characterized previous treatments of such deposition phenomena, viz. separability ("frozen" vapor BL) or local vapor/condensate equilibrium†—indeed, our treatment provides as a by-product, useful quantitative criteria for the accuracy of these earlier simplifications in terms of accessible thermophysical parameters describing the aerosol/vapor/carrier gas/target system. More generally, the present theory is readily extended to describe the effects on stagnation region condensate deposition rate of particles which: (a) are of nonuniform size; and (b) have a size-dependent thermophoretic diffusivity—important extensions which will be the subject of follow-on studies at this laboratory.

Particle-induced vapor scavenging effects in laminar boundary layers away from the forward stagnation region can also be numerically predicted with the present thermophysical model but we are then led to coupled *partial* differential equations rather than coupled *ordinary* differential equations. The present study not only provides an understanding of what can be expected in such cases,‡ but also the quantitative techniques/results needed to initiate such downstream calculations for a "blunt"-nosed collector (e.g. turbine blade or circular cylinder).

While we have not dealt here with the complicating phenomena associated with particle *inertia* [typically encountered only for supermicron particles (e.g. Fernandez de la Mora & Rosner 1981) or the nondeterministic aspects of turbulent gas and particle motion [i.e. effective particle diffusion (Rosner & Fernandez de la Mora 1982)], the insights obtained from the present analysis should also provide the necessary background against which analyses of such situations can be fruitfully undertaken. In this way we hope the present theory contributes in a systematic way to the understanding and ultimate control of undesirable or desirable deposition phenomena encountered in the materials processing industries and in many energy conversion applications.

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## NOMENCLATURE

$A$  = Function defined by [33]

$B$  = Function defined by [34]

$C$  = Damköhler parameter (for nonequilibrium vapor scavenging) defined by [35]

†Analogous phenomena occur in the partitioning of dopant elements between the vapor phase and glass particulate phase in the deposition of optical waveguide preforms (Park & Rosner 1988).

‡Indeed, we have recently shown that the observed effects of submicron MgO(s) particles on reducing the deposition rate of Na<sub>2</sub>SO<sub>4</sub> (Rosner & Liang 1988) in a laboratory combustion environment correlated with the scavenging particle surface area per unit volume.

$D_v$  = Vapor diffusion coefficient (Fick)

$f$  = Modified stream function, [10]

$h$  = Normalized total particle radius (including condensate),  $h \equiv \frac{r}{r_\infty}$

$j_w''$  = Local mass flux at the wall

$\mathcal{J}$  = Dimensionless deposition rate, [48] and [51]

$K$  = Kelvin parameter, [40]

$L$  = Mainstream particle loading parameter defined by [36]

$\mathcal{L}$  = Vaporization heat parameter defined by [42]

$M_v$  = Vapor molecular weight

$n$  = Particle number density ratio,  $n \equiv \frac{N_p}{N_{p,\infty}}$

$N_p$  = Particle number density

$p_v$  = Vapor pressure

$Pr$  = Prandtl number of the carrier gas, diffusivity ratio =  $\frac{\nu}{\alpha_h}$

$r$  = Particle radius

$\dot{r}'''$  = Local mass rate of vapor consumption per unit volume, [29]

$R$  = Universal gas constant

$s$  = Saturation ratio,  $s \equiv \frac{\Omega_v}{[\Omega_v^{\text{eq}}(\theta, h = \infty)]}$

$Sc$  = Vapor Schmidt number in the mixture, diffusivity ratio =  $\frac{\nu}{D_v}$

$t$  = Time

$T$  = Local temperature of the gas mixture

$u_e$  = External (potential) flow velocity in the  $x$ -direction [6]

$\mathbf{v}$  = Velocity field ( $u, v$ )

$x$  = Distance along the wall (measured from the stagnation point)

$X$  = Defined by [62]

$y$  = Distance normal to the wall

$Y$  = Defined by [63]

### Greek symbols

$\alpha$  = "Normalized" thermophoretic diffusivity for particle migration,  $\alpha \equiv \frac{\alpha_T D_p}{\nu}$

$\alpha_h$  = Heat (thermal) diffusivity of the carrier gas

$\alpha_m$  = Mass accommodation (condensation) coefficient, [2]

$\alpha_T$  = Thermal diffusion factor for a condensate covered particle, [1]

$\Gamma$  = Function defined by [56]

$\delta_T$  = Function defined by [21]

$\delta_v$  = Function defined by [57]

$\eta$  = Dimensionless distance from the wall, [9]

$\theta$  = Dimensionless absolute temperature field,  $\theta \equiv \frac{T}{T_\infty}$

$\Lambda$  = Constant in [5] (molar heat of vaporization of macroscopic condensate)

$\nu$  = Gas momentum diffusivity (kinematic viscosity)

$\rho$  = Mass density of mixture

$\rho_c$  = Condensate "phase" mass density (condensate mass per unit volume of total space)

$\sigma$  = Surface tension of the condensate

$\psi$  = Stream function, [10]

$\omega_v$  = Vapor mass fraction,  $\omega_v \equiv \frac{\rho_v}{\rho}$

$\omega_c$  = Condensate mass fraction,  $\omega_c \equiv \frac{\rho_c}{\rho}$

$\Omega$  = Vapor mass fraction normalized with respect to  $\omega_{v,\infty}$

$\hat{\Omega}_v''$  = Nondimensional local vapor consumption rate, [53]

#### Subscripts

c = Corresponding to the condensate (referred to the total volume)

dp = Dew point

L = Corresponding to the condensate (liquid)

hom nucl = Homogeneous nucleation

p = Corresponding to the particles

T = Thermophoretic

v = Corresponding to the vapor

w = At the wall (deposition surface)

$\infty$  = At mainstream

#### Superscripts

eq = Thermodynamic equilibrium value (VCE)

' = Derivative with respect to  $\eta$

#### Abbreviations

BL = Boundary layer

div = Divergence (operator)

FBL = "Frozen" boundary layer (no phase change)

grad = Spatial gradient (operator)

LTE = Local thermodynamic equilibrium (VCE)

ODE = Ordinary differential equation

PDE = Partial differential equation

r.h.s. = Right-hand side

VCE = Vapor/condensate equilibrium

## REFERENCES

- AHLUWALIA, R. K. & IM, K. H. 1985 Mass transfer of trace species from a superheated stream to a subcooled surface. *Int. J. Heat Mass Transfer* **28**, 2061–2070
- CASTILLO, J. L. & ROSNER, D. E. 1988a Theory of surface deposition from a unary dilute vapor-containing stream allowing for condensation within the laminar boundary layer. *Chem. Engng Sci.* In press.
- CASTILLO, J. L. & ROSNER, D. E. 1988b Theory of surface deposition from particle-laden dilute, saturated vapor-containing streams, allowing for particle thermophoresis and vapor scavenging within the laminar boundary layer. *Chem. Engng Sci.* In press.
- CASTILLO, J. L. & ROSNER, D. E. 1988c Theory of surface deposition from a binary dilute vapor-containing stream allowing for equilibrium condensation within the laminar boundary layer. *Int. J. Multiphase Flow.* In press.
- EPSTEIN, M. & ROSNER, D. E. 1970 Enhancement of diffusion-limited vaporization rates by condensation within the thermal boundary layer. 2. Comparison of homogeneous nucleation theory with the critical supersaturation model. *Int. J. Heat Mass Transfer* **13**, 1393–1414.
- FERNANDEZ DE LA MORA, J. & ROSNER, D. E. 1981 Inertial deposition of particles revisited and extended: Eulerian approach to a traditionally Lagrangian problem. *Physicochem. Hydrodynam.* **2**, 1–21.
- GÖKOĞLU, S. A. & ROSNER, D. E. 1984 Engineering correlations of variable-property effects on laminar forced convection mass transfer for dilute vapor species and small particles in air. Report NASA CR-168322.
- GÖKOĞLU, S. A. & ROSNER, D. E. 1985 Thermophoretically enhanced mass transport rates to solid and transpiration-cooled walls across turbulent (law-of-the-wall) boundary layers. *Ind. Engng Chem. Fundam.* **24**, 208–214.

- GOREN, S. L. 1977 Thermophoresis of aerosol particles in the laminar boundary layer on a flat plate. *J. Colloid Interface Sci.* **61**, 77–85.
- JANZ, G. J., ALLEN, C. B., BANSAL, N. P., MURPHY, R. M. & TOMKINS, R. P. T. 1979 Physical properties data compilations relevant to energy storage. II Molten salts: data on single and multi-component salt systems. Report NSRDS-NBS No. 61, Part II.
- LIANG, B., GOMEZ, CASTILLO, J. L. & ROSNER, D. E. 1988 Experimental studies of nucleation phenomena within thermal boundary layers—influence on chemical vapor deposition rate processes. Presented at 1987 A. Mtg American Association for Aerosol Research, Seattle, Wash., Paper A5A. *AICHE Jl.* Submitted.
- PARK, H. M. 1987 Thermophoretically augmented mass-, momentum- and energy transfer rates in high particle mass loaded laminar forced convection systems. Ph.D. Dissertation, Dept of Chemical Engineering, Yale Univ., New Haven, Conn.
- PARK, H. M. & ROSNER, D. E. 1988 Dopant redistribution across aerosol-laden laminar non-isothermal boundary layers. *Chem. Engng Sci.* In press.
- ROSNER, D. E. 1980 Thermal (Soret) diffusion effects on interfacial mass transport rates. *Physicochem. Hydrodynam.* **1**, 159–185.
- ROSNER, D. E. 1986 *Transport Processes in Chemically Reacting Flow Systems*. Butterworths, Stoneham, Mass.
- ROSNER, D. E. & ATKINS, R. M. 1983 Experimental studies of salt/ash deposition rates from combustion products using optical techniques. In *Fouling and Slagging Resulting from Impurities in Combustion Gases* (Edited by BRYERS, R.), pp. 469–492. Engng Foundation, New York.
- ROSNER, D. E. & FERNANDEZ DE LA MORA, J. 1982 Small particle transport across turbulent non-isothermal boundary layers. *ASME Trans. J. Engng Power* **104**, 885–894.
- ROSNER, D. E. & KIM, S. S. 1984 Optical experiments on thermophoretically augmented submicron particle deposition from 'dusty' high temperature gas flows. *Chem. Engng J.* **29**, 147–157.
- ROSNER, D. E. & LIANG, B. 1988 Experimental studies of deposition rates in the presence of alkali sulfate vapor scavenging by submicron particles in combustion gas boundary layers. *Chem. Engng Commun.* In press. See also ROSNER, D. E. 1988 Experimental and theoretical studies of the deposition of inorganic compounds from combustion gases. *J. electrochem. Soc.* Submitted.
- ROSNER, D. E. & PARK, H. M. 1988 Thermophoretically augmented mass, momentum and energy transfer rates in high particle mass-loaded laminar forced convection systems. *Chem. Engng. Sci.* In press.
- ROSNER, D. E., CHEN, B., FRYBURG, G. C. & KOHL, F. J. 1979 Chemically frozen multicomponent boundary layer theory of salt and/or ash deposition rates from combustion gases. *Combust. Sci. Technol.* **20**, 87–106.
- ROTHMAN, M. F. (Ed.) 1985 *High Temperature Corrosion in Energy Systems*. The Metallurgical Society of AIME.
- SCHLICHTING, H. 1968 *Boundary Layer Theory*, 6th edn. McGraw-Hill, New York
- SPALDING, D. B. & EVANS, M. L. 1961 Mass transfer through laminar boundary layers—3. Similar solutions of the b-equation. *Int. J. Heat Mass Transfer* **2**, 314–341.
- TALBOT, L. 1981 Thermophoresis—a review. In *Rarefied Gas Dynamics*, Part I (Edited by FISHER, S. S.). *Prog. Astronaut. Aeronaut.* **74**, 467–488.
- WALKER, K. L., HOMSY, G. M. & GEYLING, F. T. 1979 Thermophoretic deposition of small particles in laminar tube flow. *J. Colloid Interface Sci.* **69**, 138–147.

## APPENDIX

In this appendix we restrict ourselves to the case  $K = 0$ . As indicated in section 4, when the product  $CL$  is large, the present model must be refined because of the addition of a third region around the cold surface besides the two regions already considered in detail in section 3.1. Each region would be characterized by the following properties.

Region 1 (undersaturated) for  $\eta > \eta_n$  (as indicated at the beginning of section 3.2), where  $\Omega_v < \Omega_v^{\text{eq}}$  and the vapor cannot condense on the particles.

Region 2 (supersaturated) for  $\eta_n \geq \eta \geq \eta^{eq}$  (section 3.2), where  $s = 1$  only at the boundaries of the region and  $s > 1$  elsewhere. Here the particles scavenge as much vapor as they can, the governing equations are [64]–[66] with the boundary conditions [67]–[69] at  $\eta = \eta_n$ .

Region 3 (equilibrium) for  $\eta^{eq} \geq \eta \geq 0$ , in which  $s = 1$  everywhere. Here the particles capture as much vapor as is available, i.e. there is equilibrium-limited vapor scavenging. The vapor field equation for this third region is

$$\Omega_v(\eta) = \Omega_v^{eq}(\theta(\eta)), \quad [\text{A.1}]$$

where the dependence on the radius ratio  $h$  has been omitted in  $\Omega_v^{eq}$  because we are considering the case  $K = 0$  (negligible Kelvin effect). Using this value of  $\Omega_v$  in [32] together with [18], the local vapor consumption rate  $\dot{\Omega}_v''$  defined by [53] becomes

$$\dot{\Omega}_v''(\eta) = \frac{1}{Sc} \frac{d^2 \Omega_v^{eq}}{d\theta^2} \left( \frac{d\theta}{d\eta} \right)^2 + f \cdot \left( 1 - \frac{Pr}{Sc} \right) \frac{d\Omega_v^{eq}}{d\theta} \cdot \frac{d\theta}{d\eta}, \quad [\text{A.2}]$$

with  $\Omega_v^{eq}(\theta)$  given by [39] and  $\theta(\eta)$  by [20]. With this vapor consumption the equation for the normalized particle radius  $h$  becomes

$$A \frac{dh}{d\eta} = -\frac{\dot{\Omega}_v''}{3Lnh^2}. \quad [\text{A.3}]$$

At the “interface” between the second and the third regions the following conditions should be satisfied:

$$\Omega_v = \Omega_v^{eq}(\theta(\eta^{eq})), \quad \eta = \eta^{eq}, \quad [\text{A.4}]$$

and

$$\frac{d\Omega_v}{d\eta} = \frac{d\Omega_v^{eq}}{d\theta} \cdot \frac{d\theta}{d\eta} \quad @ \quad \eta = \eta^{eq} \quad [\text{A.5}]$$

and, also, the value of  $h$  should be continuous at  $\eta = \eta^{eq}$ . These conditions allow us to specify the positions of both  $\eta_n$  and  $\eta^{eq}$ . Using [A.1] and [39] the deposition of condensible material in vapor form,  $\mathcal{J}_v$ , becomes

$$\mathcal{J}_v = \frac{1 - \theta_w}{Sc \delta_T \theta_w^2} \cdot \theta_{dp} \cdot \left( \frac{\mathcal{L}}{\theta_w} - 1 \right) \cdot \exp \left[ \mathcal{L} \left( \frac{1}{\theta_{dp}} - \frac{1}{\theta_w} \right) \right]. \quad [\text{A.6}]$$

Observe that this is the same as the result corresponding to the LTEBL limit. The deposition rate of material condensed on the particles is still governed by [52].

In our calculation we have found that the value of  $\eta_n$  decreases as  $C$  or  $L$  increases (keeping the other constant) and, for sufficiently large values of  $CL$  when the third region appears, the value of  $\eta^{eq}$  should increase with  $C$  or  $L$ . Therefore, for a certain finite value of  $CL$  the limit in which  $\eta_n = \eta^{eq}$  (and the second supersaturated region disappears) would be reached. The LTEBL limit would have been reached and a further increase in  $C$  or  $L$  will have no effect at all on the deposition rate results because local vapor consumption by the particles would then be equilibrium limited [cf. the case studied earlier by Castillo & Rosner (1988a)].